

Separable	$f(x) = g(y) \frac{dy}{dx}$	Separate $\frac{dy}{dx}$ and then integrate
Linear	$y' + a(x)y = b(x)$	Multiply by $e^{A(x)}$ where $A(x) = \int a(x)dx$
Exact	$M(x, y) + N(x, y) \cdot y' = 0$	Test: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Find f where $\frac{\partial f}{\partial x} = M$ and $\frac{\partial f}{\partial y} = N$ Solution: $f(x, y) = c$
Constant coefficient	$a_2y'' + a_1y' + a_0y = 0$	Find roots of characteristic equation If two real roots r_1, r_2 use e^{r_1x} and e^{r_2x} If double real root r use e^{rx} and xe^{rx} If complex roots $r = \alpha \pm \beta i$ use $e^{\alpha x} \cos(\beta x)$ and $e^{\alpha x} \sin(\beta x)$
Variation of parameters	$y'' + a_1(x)y' + a_0(x)y = b(x)$	$y_p = v_1y_1 + v_2y_2$ $v_1 = \int \frac{-y_2 \cdot b(x)}{W(y_1, y_2)} dx$ $v_2 = \int \frac{y_1 \cdot b(x)}{W(y_1, y_2)} dx$

Undetermined coefficients guess for y_p :

$b(x)$	y_p
constant	A
$5x - 3$	$Ax + B$
$10x^2 - x + 1$	$Ax^2 + Bx + C$
$\sin(6x)$	$A \cos(6x) + B \sin(6x)$
$\cos(6x)$	$A \cos(6x) + B \sin(6x)$
e^{3x}	Ae^{3x}
$(2x + 1)e^{3x}$	$(Ax + B)e^{3x}$
x^2e^{3x}	$(Ax^2 + Bx + C)e^{3x}$
$e^{3x} \sin(4x)$	$Ae^{3x} \cos(4x) + Be^{3x} \sin(4x)$
$e^{3x} \cos(4x)$	$Ae^{3x} \cos(4x) + Be^{3x} \sin(4x)$
$5x^2 \sin(4x)$	$(Ax^2 + Bx + C) \cos(4x) + (Dx^2 + Ex + F) \sin(4x)$

Taylor series: $f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x-x_0)^3 + \frac{f^{(4)}(x_0)}{4!}(x-x_0)^4 + \dots$

$$\int u dv = uv - \int v du \quad \int \sin(x) dx = -\cos(x) \quad \int \cos(x) dx = \sin(x)$$